ANALYSIS OF TE10 MODE PROPAGATION IN AN AIR-FILLED VS. A DIELECTRICALLY-FILLED (ALUMINA) RECTANGULAR WAVEGUIDE

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Abstract

This project presents a comparative analysis of the dominant TE_{10} mode propagation in a standard air-filled rectangular waveguide versus a waveguide fully-filled with 96% Alumina (Al2O3). The study utilizes the finite element method (FEM) within the Ansys HFSS simulation software to model and analyze both homogeneous configurations. Key propagation characteristics—including cutoff frequency, electric and magnetic field distributions, guided wavelength, and insertion loss—are extracted, compared, and validated against their theoretical analytical formulas.

The results demonstrate that filling the guide with alumina significantly lowers the TE_{10} cutoff frequency (from ~6.56 GHz to ~2.19 GHz), enabling propagation at frequencies where the air-filled guide is cutoff. Furthermore, the analysis shows a distinct concentration of the electric and magnetic fields, a guided wavelength shortened by approximately a factor of three, and the introduction of measurable dielectric losses (due to $\tan\delta$) not present in the ideal air guide. This study provides a quantitative understanding of the fundamental trade-offs of 100% dielectric loading; a technique used for component miniaturization.

Keywords

Rectangular Waveguide, Dielectric Loading, Alumina, TE_{10} Mode, Propagation Characteristics, Ansys HFSS, Finite Element Method (FEM), Cutoff Frequency, Guided Wavelength, Field Distribution, Dielectric Loss, Microwave Engineering, Comparative Analysis.

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Introduction

1.1 General Context and Motivation

Rectangular waveguides are a foundational component in microwave engineering, providing a low-loss and high-power handling medium for guiding electromagnetic (EM) energy. They are indispensable in systems ranging from radar and particle accelerators to satellite communications. As modern RF systems demand higher frequencies, wider bandwidths, and smaller physical footprints, the modification of simple, air-filled guides becomes a critical design technique.

One primary technique for achieving component miniaturization and altering propagation characteristics is dielectric loading. By inserting a dielectric material, such as alumina, into the waveguide, the EM field distribution and boundary conditions are changed. This technique is the basis for many practical devices, including phase shifters, filters, and resonators, but it introduces a new set of design trade-offs.

1.2 Problem Statement

While the behavior of the dominant TE_{10} mode in a homogeneous, air-filled waveguide is well-understood, the "problem" for a design engineer is to accurately predict and quantify the trade-offs of modifying this structure.

Fully loading the guide with a high-permittivity dielectric, such as 96% Alumina, creates a new, homogeneous guide with vastly different characteristics. It is not intuitively obvious how the cutoff frequency, guided wavelength, impedance, and signal loss will be affected.

A clear, quantitative comparison is required to understand the fundamental trade-offs between miniaturization and performance (loss). This project directly compares these two extreme cases: 0% dielectric loading (Air) and 100% dielectric loading (Alumina).

1.3 Project Objectives and Milestones

The primary objective of this mini-project is to conduct a quantitative, simulation-based comparison between a standard air-filled waveguide and one fully loaded with an alumina (Al2O3) slab.

The specific milestones to achieve this objective are:

1. **To model** both the air-filled and alumina-loaded waveguide configurations accurately within the Ansys HFSS simulation environment.
2. **To determine and compare** the TE_{10} mode cutoff frequency (f_c) for each guide by analyzing its transmission (S_{21}) response.
3. **To visualize and analyze** the differences in the E-field and H-field distributions, specifically focusing on field concentration within the dielectric.
4. **To measure and compare** the guided wavelength (λ_g) for both configurations at a fixed operating frequency.
5. **To quantify and contrast** the insertion loss (attenuation) introduced by the alumina's dielectric properties ($\tan\delta$) versus the near-lossless air-filled guide.

1.4 Methodology and Validation Criteria

The methodology employed is computational modeling using the 3D Finite Element Method (FEM) solver within Ansys HFSS. Both waveguide models are excited at Port 1 with the TE_{10} mode and terminated at Port 2. A "Driven Modal" frequency sweep analysis is performed to extract S-parameters (S_{11} , S_{21}) and generate 3D field plots.

The validation criteria for this simulation approach are twofold:

1. **Model Validation:** The simulated results for the air-filled waveguide (the control case) will be validated against the well-established theoretical formula for its TE_{10} cutoff frequency $(f_c = c/2a)$. A strong correlation will validate the model setup, port configuration, and mesh.
2. **Result Plausibility:** The results for the alumina-loaded guide, will also be validated against its known analytical formula $\left(\boldsymbol{f_c' = c/(2a\sqrt{\epsilon_r})}\right)$. This provides a second, powerful validation check. All other results (field concentration, shorter λ_g, loss) will be assessed for physical plausibility.

1.5 Report Structure

This report is structured to logically present the findings. **Chapter 2** details the theoretical background of waveguide propagation and dielectric properties. **Chapter 3** describes the simulation setup and parameters used in Ansys HFSS. **Chapter 4** presents the core of the project: a direct comparative analysis of the simulation results. Finally, **Chapter 5** concludes the findings and summarizes the practical impact of dielectric loading.

Chapter 2: Theoretical Background

2.1 The Rectangular Waveguide

2.1.1 Definition and Function

A rectangular waveguide is a high-frequency transmission line consisting of a hollow, metallic tube with a rectangular cross-section. Its primary function is to guide electromagnetic (EM) waves, typically in the microwave spectrum (GHz), from one point to another.

Unlike coaxial cables, a waveguide has no central conductor. The wave propagates within the hollow space, confined by the four conductive walls. These walls act as a boundary, forcing the EM fields to conform to specific patterns (known as "modes") to propagate. The guide's metallic structure, typically made of copper or aluminum, is often treated as a **Perfect Electrical Conductor (PEC)** in ideal analysis.

2.1.2 Key Advantages

Waveguides are foundational in high-frequency applications like radar, satellite communications, and particle accelerators due to two major advantages over other transmission lines:

- **High Power Handling:** The absence of a small central conductor (which can overheat) and the large volume allow waveguides to transmit extremely high power levels without dielectric breakdown or thermal failure.
- **Low Attenuation (Loss):** The wave propagates primarily through the low-loss dielectric filling the guide (in this project, air or alumina). This results in significantly lower signal loss (attenuation) per unit length compared to coaxial cables, especially at frequencies above a few GHz.

2.2 Electromagnetic Modes (TE & TM)

2.2.1 The Concept of Modes

An electromagnetic wave cannot propagate inside a hollow waveguide in any arbitrary form. The conductive boundary walls force the electric field (E) to be perpendicular and the magnetic field (H) to be parallel at the conductor's surface. These boundary conditions only permit the wave to travel in specific, discrete field patterns known as **"modes."**

Each mode is a unique solution to Maxwell's Equations for the given waveguide geometry and is characterized by its own cutoff frequency and field distribution. These modes are broadly classified into two families: Transverse Electric (TE) and Transverse Magnetic (TM).

2.2.2 Mode Definitions

- **TE (Transverse Electric) Modes:** In a TE mode, the **electric field (E) vector is entirely transverse** (perpendicular) to the direction of wave propagation (conventionally the z-axis). However, it has a necessary component of the magnetic field (H) *in* the direction of propagation. These modes are also sometimes called H-modes.
- **TM (Transverse Magnetic) Modes:** In a TM mode, the **magnetic field (H) vector is entirely transverse** (perpendicular) to the direction of propagation. It must have a component of the electric field (E) *in* the direction of propagation. These are also known as E-modes.

2.3 The TE_{10} Dominant Mode

2.3.1 Definition of the Dominant Mode

Every waveguide has an infinite number of possible propagation modes (e.g., $\boldsymbol{TE_{10}}$, $\boldsymbol{TE_{20}}$, $\boldsymbol{TE_{01}}$, $\boldsymbol{TM_{11}}$, etc.), each with its own cutoff frequency. The **"dominant mode"** is the mode with the **lowest cutoff frequency** ($\boldsymbol{f_c}$).

This is the most critical mode in practice. If a signal's frequency is slowly increased from zero, the dominant mode is the *first* one that can propagate. To ensure predictable behavior, waveguides are almost always operated in a frequency range where only the dominant mode can propagate. This is known as **single-mode operation**.

2.3.2 Field Structure of the TE_{10} Mode

For a standard rectangular waveguide where the wide dimension a is greater than the narrow dimension b (a > b), the dominant mode is always the $\boldsymbol{TE_{10}}$ **mode**. The "1,0" indices indicate:

- m = 1: There is one half-wavelength variation of the electric field across the wide dimension a.

- n = 0: There is zero (constant) variation of the electric field across the narrow dimension b.

The field structure is distinct:

- **Electric Field (E):** The E-field is purely transverse. It is oriented entirely along the narrow b dimension (the y-axis). Its magnitude is zero at the side walls (x=0, x=a) and reaches a single maximum in the center of the guide (x=a/2).
- **Magnetic Field (H):** The H-field has components both transverse (x-axis) and in the direction of propagation (z-axis). The field lines form complete, continuous loops that wrap around the E-field maxima in the longitudinal (x-z) plane.

2.4 Cutoff Frequency (f_c)

2.4.1 Definition

The **cutoff frequency** (f_c)is a critical parameter of a waveguide. It represents the **minimum frequency** at which a specific mode can propagate energy down the guide.

Below this frequency, the mode is said to be **"evanescent"** or "cutoff." The wave does not propagate; instead, it is rapidly attenuated (its amplitude decays exponentially) from the point of excitation. A waveguide effectively acts as a **high-pass filter** for each of its modes.

2.4.2 TE_{10} Cutoff Frequency Formula

The cutoff frequency for any TE_{mn} or TM_{mn} mode in a lossless, air-filled rectangular waveguide is given by the general formula:

$$f_{c(mn)} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Where:

- c is the speed of light in a vacuum (~ 3×10^8 m/s).
- a is the wide dimension.
- b is the narrow dimension.

- m and n are the mode indices (integers).

For the $\boldsymbol{TE_{10}}$ **dominant mode**, we set m=1 and n=0. The general formula simplifies significantly to:

$$f_{c(10)} = \frac{c}{2a}$$

Using our dimensions:

$$f_{c(\text{air})} = \frac{3 \times 10^8}{2 \times 0.02286} \approx 6.56GHz$$

$$f_{c(\text{alumina})} = \frac{3 \times 10^8}{2 \times 0.02286 \times \sqrt{9.0}} = \frac{6.56\ \text{GHz}}{3} \approx 2.19GHz$$

2.4.3 Importance for this Project

This simple, analytical formula is central to this project for two reasons:

1. **Baseline Calculation:** It provides the exact theoretical cutoff frequency for the **air-filled waveguide (control case)**.
2. **Simulation Validation:** The simulated f_c for the air-filled model *must* closely match this theoretical value. A strong agreement validates the entire simulation setup (model geometry, ports, and mesh) and provides confidence that the results for the more complex, alumina-loaded guide (which has no simple formula) are also accurate.

2.5 Dielectric Loading and Material Properties

2.5.1 The Concept of Dielectric Loading

Dielectric loading is the practice of inserting a dielectric material—an insulator with a permittivity greater than that of a vacuum—into a waveguide or resonant cavity. In this project, the guide is *partially filled* with an alumina slab, creating a non-homogeneous (or heterogeneous) structure where the wave propagates through both air and alumina.

This technique is a fundamental tool in microwave design. As the electromagnetic fields interact with the dielectric, their structure and propagation characteristics are altered. This effect is

leveraged to achieve specific engineering goals, such as component miniaturization, wave velocity control (phase shifting), or filter design.

2.5.2 Key Material Properties

The extent to which the dielectric loading affects the wave is governed by the material's complex permittivity. For this analysis, we are concerned with two primary properties that are defined as inputs in the Ansys HFSS simulation.

- **Relative Permittivity (ϵ_r)**

Also known as the dielectric constant, this is a dimensionless, real-valued number that quantifies a material's ability to store electrical energy compared to a vacuum ($\epsilon_r = 1.0$). A higher permittivity indicates that a material will "pull" the electric field, causing more of the wave's energy to be concentrated within it. This property is the primary cause of the shift in cutoff frequency and the shortening of the guided wavelength.

- **Loss Tangent ($\tan \delta$)**

This is a dimensionless, real-valued number that quantifies the "lossiness" of a dielectric material. It represents the ratio of dissipated power (lost as heat) to stored power within the material. An ideal, lossless material (like a vacuum or the "Air" in our control case) has $\tan \delta = 0$. Any real-world material, including alumina, will have a small, non-zero loss tangent. This property is the direct cause of dielectric loss (attenuation) as the wave propagates.

2.6 Key Performance Metrics

To quantify the differences between the air-filled and the alumina-loaded waveguide, two key performance metrics will be analyzed. These metrics are a direct consequence of the material properties (ϵ_r and $\tan \delta$) defined in the previous section.

2.6.1 Guided Wavelength (λ_g)

- **Definition:** The **guided wavelength (λ_g)** is the spatial period of the propagating wave *inside* the waveguide. It represents the physical distance along the direction of propagation (the z-axis) between two consecutive points of identical phase.

- **Key Concept:** This guided wavelength is *not* the same as the free-space wavelength ($\lambda_0 = c/f$). In an air-filled guide, λ_g is always *longer* than λ_0. It is a function of the operating frequency, the cutoff frequency, and the permittivity of the material filling the guide.
- **Relevance to Project:** A primary and directly observable effect of loading the guide with a high-permittivity material like alumina is the **reduction or shortening** of the guided wavelength. This phenomenon is a key principle behind component miniaturization, as it allows for resonant structures (like filters or antennas) to be physically smaller while operating at the same frequency.

2.6.2 Attenuation (Loss)

- **Definition: Attenuation (α)** is the gradual decrease in the power or amplitude of the signal as it propagates along the length of the waveguide. It is typically expressed in decibels per meter (dB/m).
- **Types of Loss:** In a practical waveguide, loss is primarily caused by two mechanisms:
    1. **Conductor Loss (α_c):** Energy lost as heat in the (imperfectly) conductive metal walls due to the induced currents.
    2. **Dielectric Loss (α_d):** Energy lost as heat within the dielectric material itself, caused by its non-zero **Loss Tangent**.
- **Relevance to Project:** By modeling the waveguide walls as a Perfect Electrical Conductor (PEC), this project intentionally isolates and analyzes only the **dielectric loss.** The ideal air-filled guide ($\tan \delta = 0$) will serve as a lossless baseline. The measured attenuation in the alumina-loaded guide will therefore be a direct quantification of the power dissipated by the alumina.

2.7 S-Parameters (Scattering Parameters)

2.7.1 Definition

At microwave frequencies, it is often impractical to measure total voltages and currents. Instead, the behavior of a component is described by **Scattering Parameters (S-parameters)**. S-

parameters relate the power waves that are *incident* on a component's ports to the power waves that are *reflected* (or "scattered") from them.

For this project, the waveguide is analyzed as a two-port network, with Port 1 as the input and Port 2 as the output. Two specific S-parameters are essential for this analysis.

2.7.2 Key Parameters for this Project

- **S_{11} (Return Loss):**

This parameter quantifies the ratio of power reflected from Port 1 to the power incident upon Port 1. It is a critical measure of how well the waveguide (and the TE_{10} mode) is "matched" to the source. A low S_{11} value (e.g., -10 dB or less) indicates a good impedance match and that very little power is being reflected.

- **S_{21} (Insertion Loss):**

This parameter quantifies the ratio of power transmitted from Port 1 to Port 2. This is the single most important metric for this comparative study, as its frequency-dependent graph will be used to determine two key characteristics:

1. **Cutoff Frequency (f_c):** The S_{21} plot will clearly identify the cutoff frequency. Below this frequency, the S_{21} value will drop sharply (to a very large negative dB value), indicating that the signal is no longer propagating through the guide.
2. **Attenuation (α_d):** In the passband (frequencies *above* f_c), any deviation from the ideal 0 dB (perfect transmission) represents the total signal loss. This value directly quantifies the **dielectric loss** introduced by the alumina's non-zero $\tan\delta$.

Chapter 3: Simulation Model and Setup

3.1 Geometric Model

This study is based on two distinct models, both configured within the Ansys HFSS simulation environment. The baseline structure for both models is a standard **WR-90 (X-Band)** rectangular waveguide.

3.1.1 Waveguide Dimensions

The internal dimensions of the WR-90 guide were set according to the EIA standard:

- **Wide Dimension (a):** 22.86 mm
- **Narrow Dimension (b):** 10.16 mm
- **Modeled Length (L):** 50 mm

3.1.2 Model A: Air-Filled (Control Case)

This study is based on two models, both configured within the Ansys HFSS simulation environment. The baseline structure for both models is a standard WR-90 (X-Band) rectangular waveguide.

3.1.3 Model B: Alumina-Filled (Test Case)

The second model is geometrically identical to Model A. However, its entire internal volume is filled with "Alumina" instead of air. This represents a homogeneous, 100% dielectrically-loaded waveguide.

3.2 Material Properties

The material properties for the simulation were defined as follows. The waveguide walls were modeled as a **Perfect Electrical Conductor (PEC)** in both cases. This is a standard assumption that isolates the analysis, ensuring that any measured attenuation is caused *only* by the dielectric fill and not by conductor (ohmic) losses in the walls.

The dielectric properties for the two models were based on standard, published values at microwave frequencies (e.g., X-Band).

Component	Material	ϵr (Permittivity)	tanδ (Loss Tangent)	Source/Justification
Waveguide Walls	PEC	N/A	N/A	Ideal conductor (lossless)
Model A Fill	Air (Ideal)	1.0	0	Lossless baseline
Model B Fill	**Alumina (96%)**	**9.0**	**0.0002**	Standard value for 96% Alumina

Table 1: Material Properties for Simulation Models

3.3 Analysis Setup

To extract the S-parameters and visualize the fields, a "Driven Modal" solution was implemented in Ansys HFSS. This solution type is designed for analyzing frequency-dependent behavior in guided-wave structures.

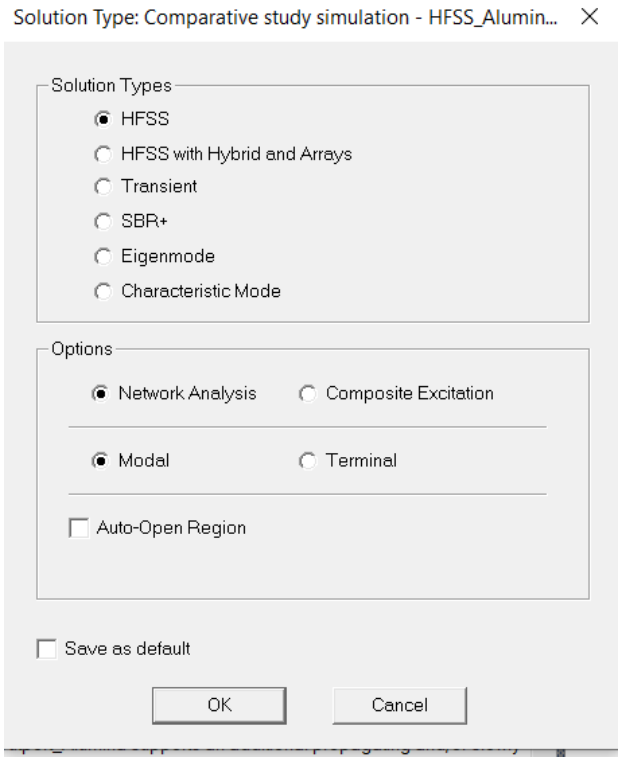

Figure 1: Driven Modal solution type

3.3.1 Frequency Sweep

A frequency sweep was set to run from **6 GHz to 15 GHz**. This range was selected for several key reasons:

1. **Baseline Validation:** It starts just below the theoretical **6.56 GHz** cutoff frequency of the air-filled (Model A) guide, allowing for a clear simulation of its cutoff behavior.
2. **Passband Coverage:** The range fully encompasses the standard operating X-band (8.2-12.4 GHz) for a WR-90 waveguide.
3. **Comparative Analysis:** Although the theoretical cutoff for the alumina-filled guide (Model B) is much lower (approx. 2.19 GHz), this sweep ensures both models are compared across an identical, high-frequency passband.

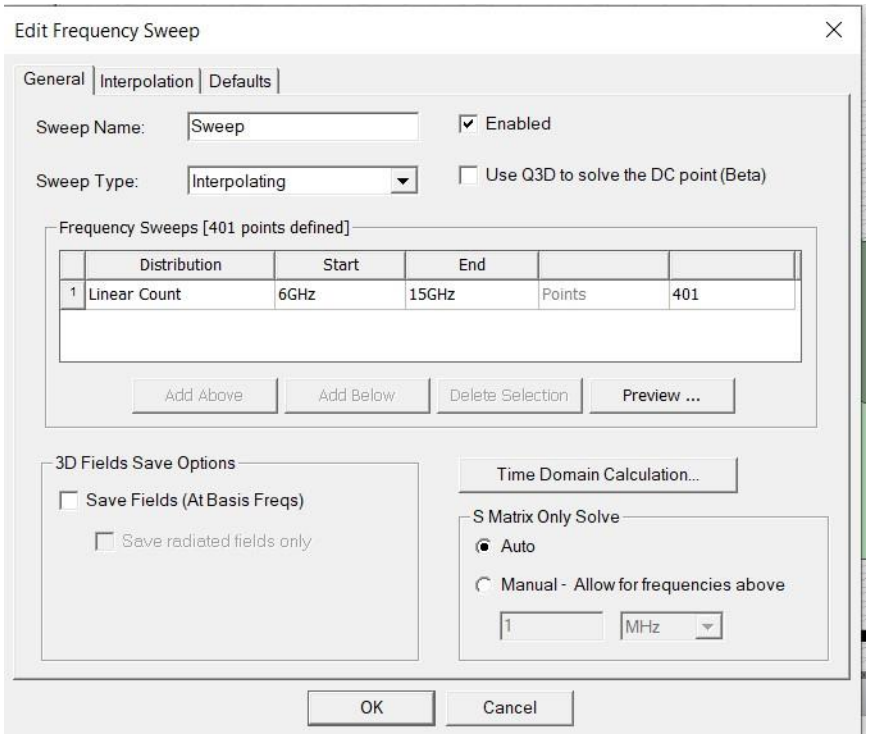

Figure 2: Frequency Sweep settings

3.3.2 Excitation

Port 1 was defined as the excitation source. It was configured to launch the fundamental $\boldsymbol{TE_{10}}$ **mode** into the waveguide. Port 2 was set as a passive, terminating load, also matched to the TE_{10} mode.

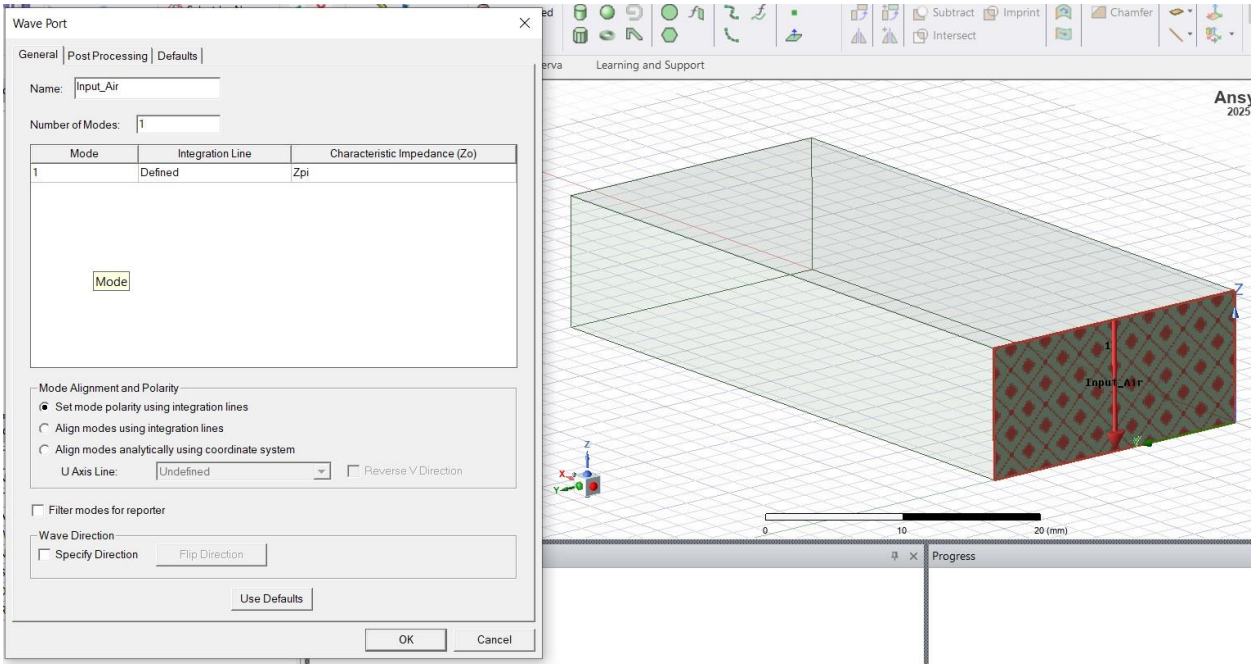

Figure 3: Input Wave Port configuration for the Air model

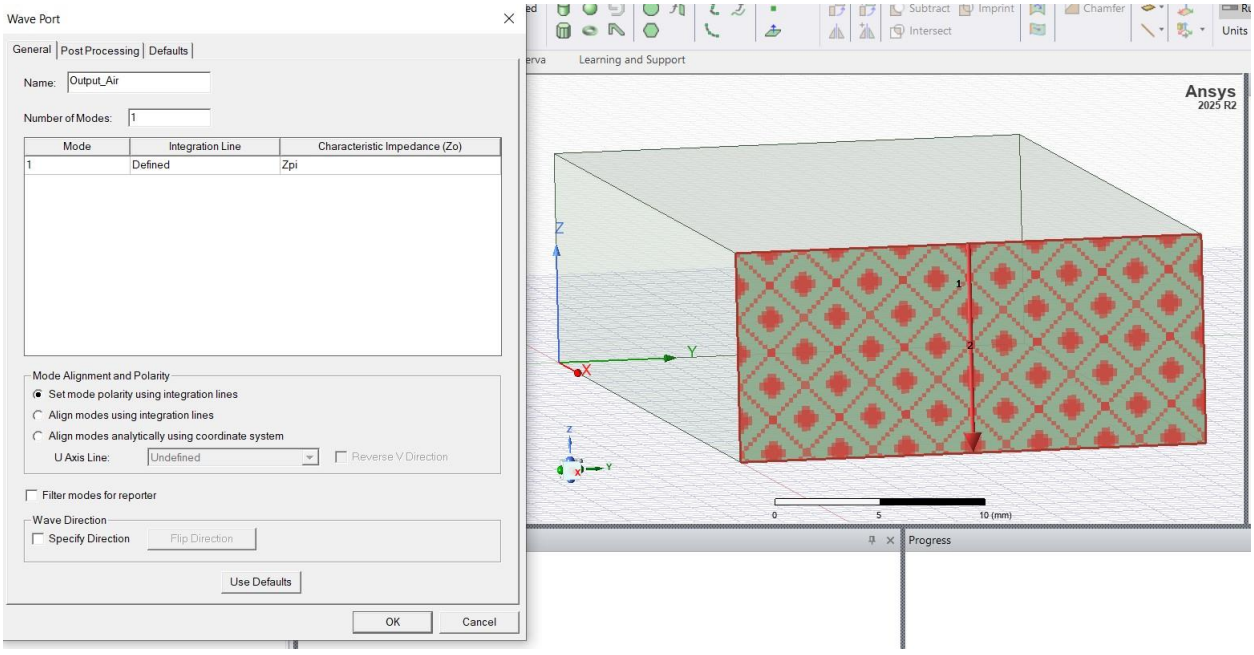

Figure 4: Output Wave Port configuration for the Air model

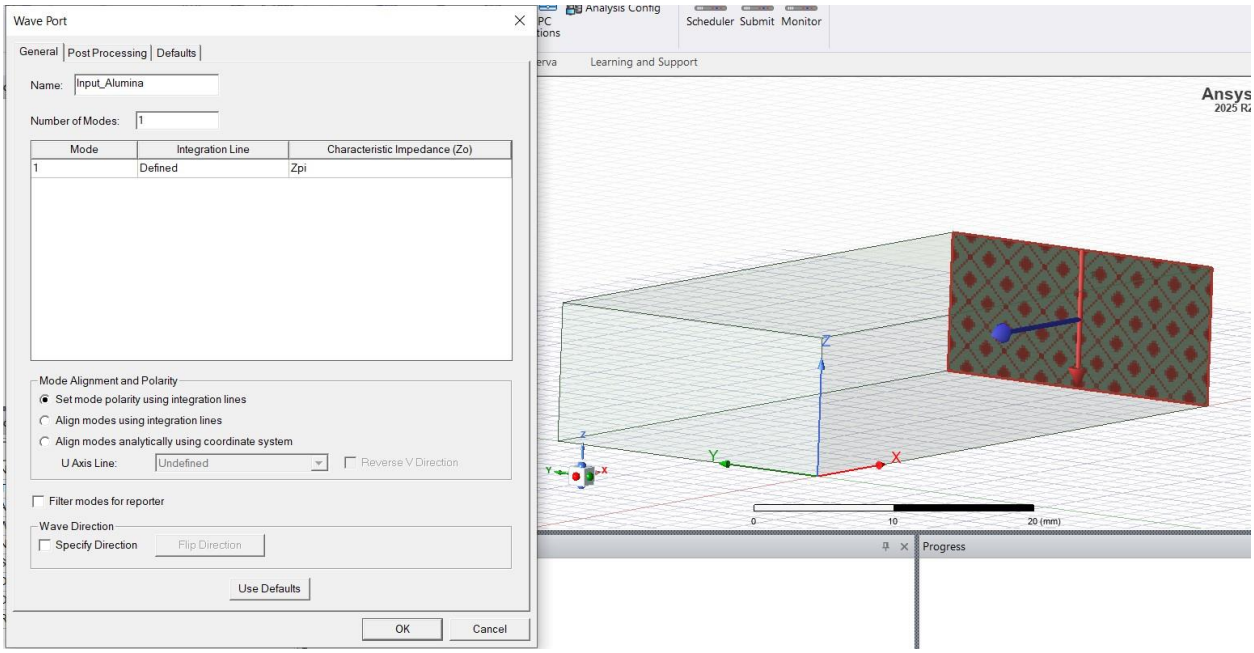

Figure 5: Input Wave Port configuration for the Alumina model

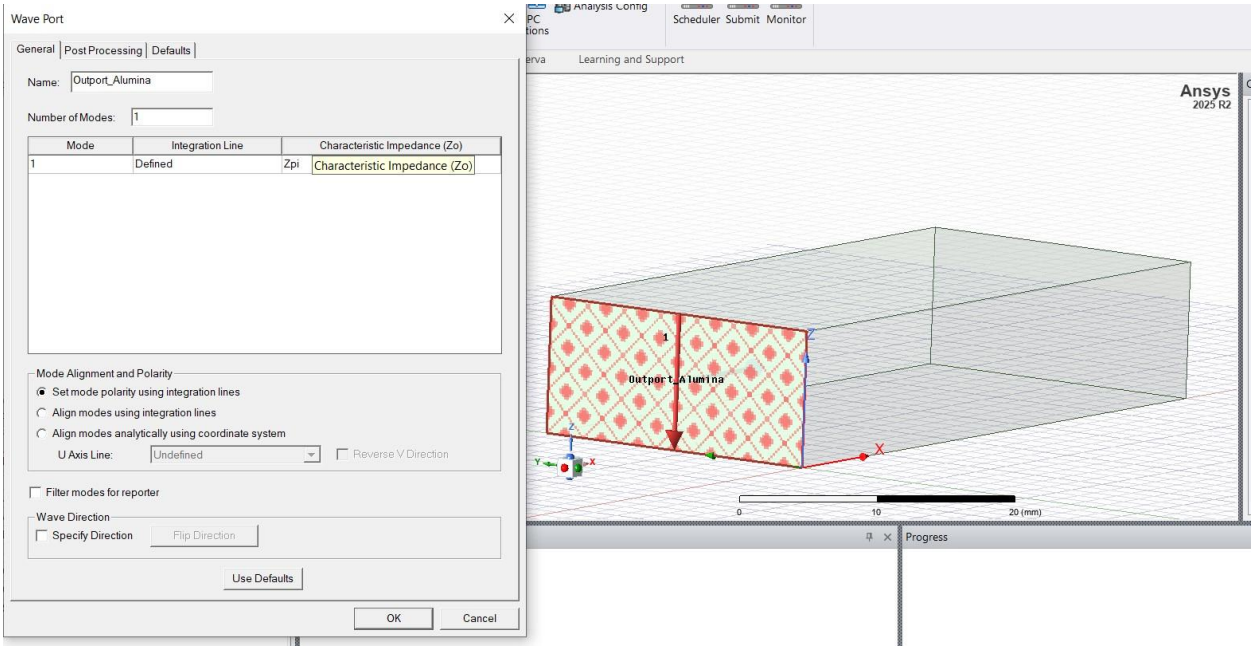

Figure 6: Output Wave Port configuration for the Alumina model

3.3.3 Analysis Frequency for Field Plots

For the direct, side-by-side comparison of E-field and H-field distributions and the measurement of guided wavelength (λ_g), a single solution frequency was set at **10 GHz**.

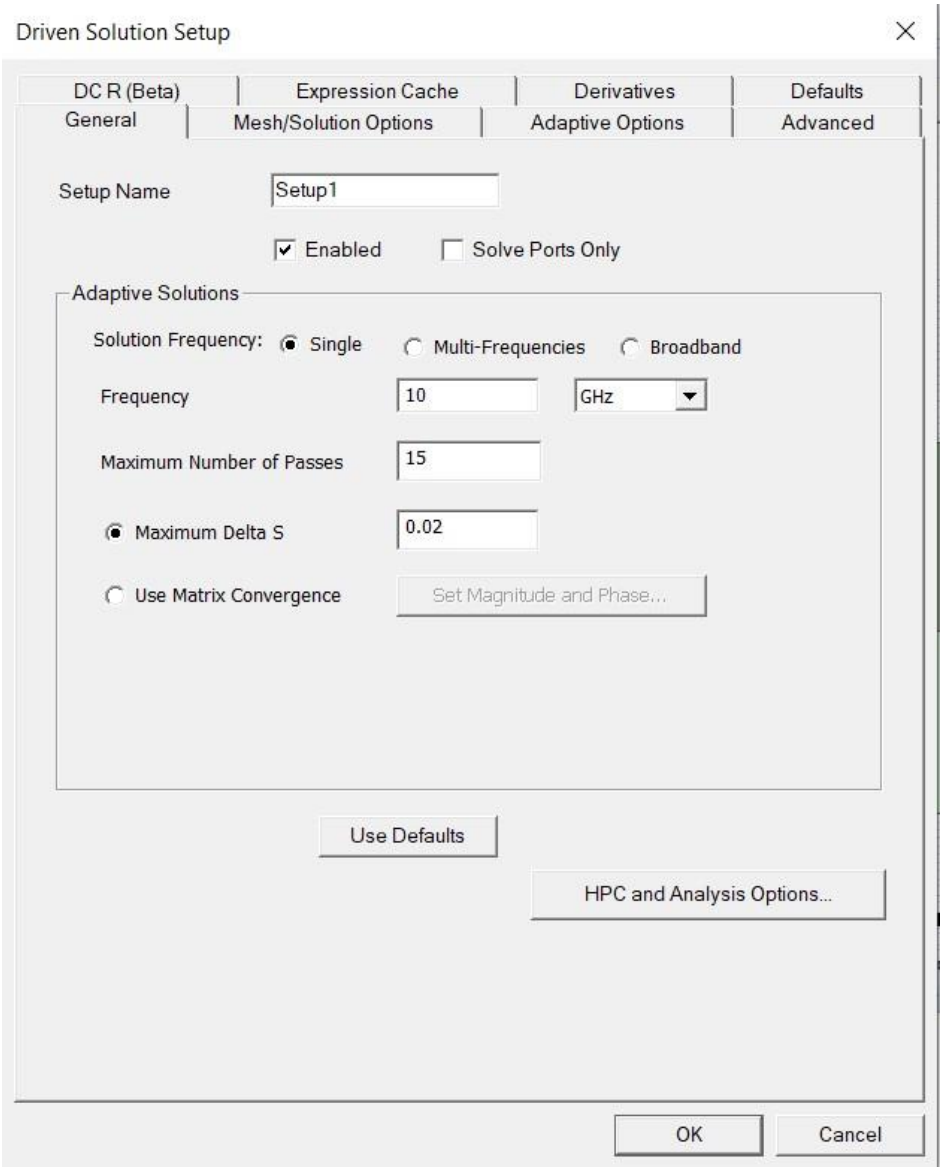

Figure 7: Driven Solution setup

This frequency is the most critical parameter for the solver's accuracy, as it serves two primary functions:

1. **Adaptive Mesh Generation:** This is the frequency HFSS uses to perform its iterative mesh refinement. By choosing 10 GHz—a representative frequency in the middle of our band of interest (X-band)—we ensure the solver generates a high-quality, converged mesh that is accurate across the entire 6-15 GHz sweep.

2. **Field Analysis Baseline:** This frequency is also the default used for all 3D field plots. 10 GHz is a standard X-band frequency and, crucially, it is confirmed to be well within the passband for *both* the air-filled model ($f_c \approx 6.56\ Hz$)and the alumina-filled model ($f_c \approx$ 2.19 GHz), allowing for a valid comparison of propagating fields.

3.3.4 Mesh and Convergence

The simulation employed the standard **adaptive mesh refinement** process. The solution was set to automatically refine the 3D mesh until the change in the S-parameters (ΔS) between two consecutive passes (iterations) was less than **0.02**. This convergence criterion ensures that the results are accurate and independent of the initial mesh.

3.4 Excitation Configuration for Field Visualization

To ensure the field plots (Chapter 4) would show a wave traveling from input to output, the post-process source settings were defined.

This configuration **does not affect the calculated S-parameter results** (which are power ratios). Instead, it sets the scaling for the visualization of 3D fields (like E-field in V/m).

For both Model A and Model B, the settings were:

- **Input Port (1:1):** Set as the active source with **1W** of power. This normalizes all field plots to a standard 1-Watt reference. It allows for a direct and fair quantitative comparison of

field intensity (e.g., "how strong is the E-field in the alumina vs. the air *for the same amount of input power*?").

- **Output Port (2:1):** Set as a passive, matched load with **0W** of power. This simulates the S_{21} (forward transmission) scenario, where the wave travels down the guide and is perfectly absorbed at the output without reflection.

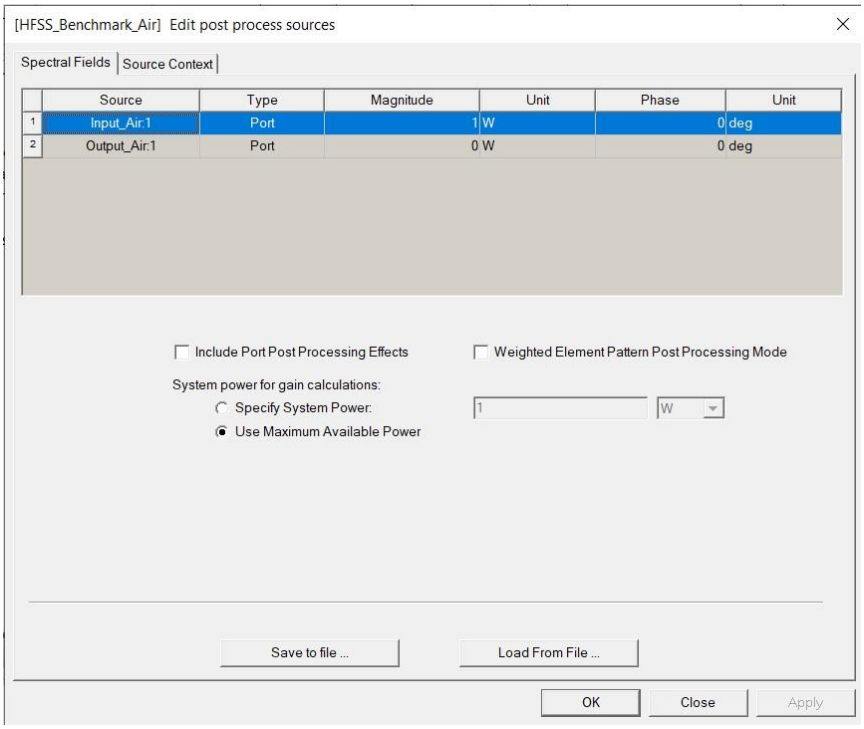

Figure 8: Post Process setup

Chapter 4: Results and Comparative Analysis

This chapter presents and analyzes the simulated data. The performance of the alumina-filled (Model B) is directly compared against the air-filled (Model A) control case to quantify the effects of dielectric loading.

4.1 Validation of the Simulation Setup

Before comparing the two models, the simulation setup must be validated using the **Model A (Air-Filled)** control case. The key metric for validation is the TE_{10} cutoff frequency.

- **Theoretical f_c:** As calculated in Chapter 2, the theoretical cutoff frequency for a WR-90 air-filled guide is **6.56 GHz**.
- **Simulated f_c:** The simulated S_{21} (transmission) for Model A is plotted below. This graph shows the insertion loss in decibels (dB) versus frequency.

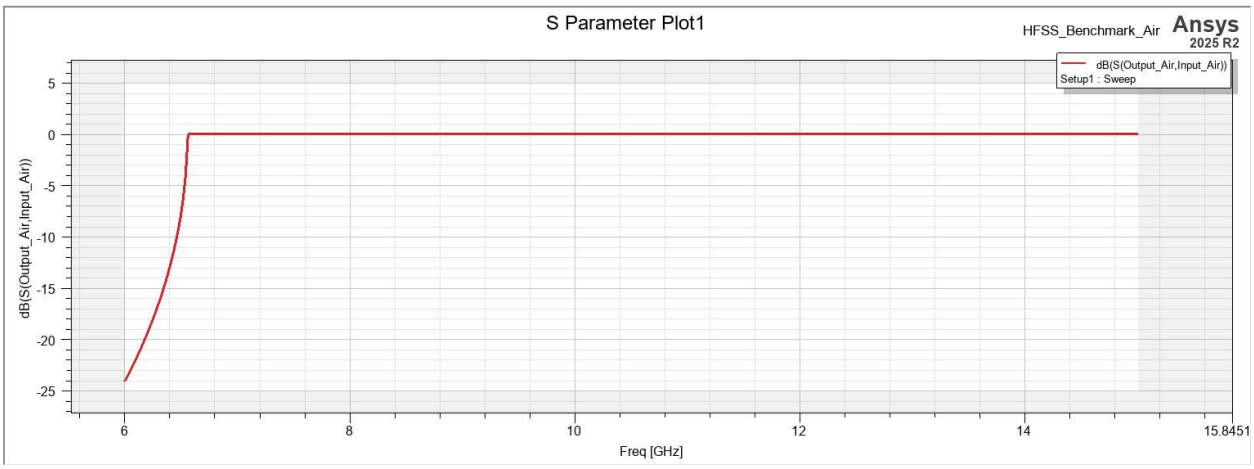

Figure 9: S_{21} (Insertion Loss) of the Air-Filled WR-90 Guide

Analysis:

The plot clearly shows the behavior of a high-pass filter. The S_{21} value is near 0 dB in the passband (indicating low loss) but drops sharply (becomes very negative) below a specific frequency. This sharp "knee" is the cutoff frequency.

From the graph, the simulated cutoff frequency is observed to be approximately **6.5 - 6.6 GHz**, which is in excellent agreement with the theoretical value of 6.56 GHz.

Conclusion:

This strong correlation between the simulated and theoretical cutoff frequencies validates the entire simulation setup (model geometry, port configuration, boundaries, and mesh). The simulation is accurate and can be trusted for the comparative analysis.

4.2 S-Parameter Analysis: Model B (Alumina-Filled)

Now that Model A (Air-Filled) has been validated as our baseline, we analyze the performance of Model B (Alumina-Filled). Figure 10 shows the overall S-parameter response (S_{11} and S_{21}) for the alumina-filled guide.

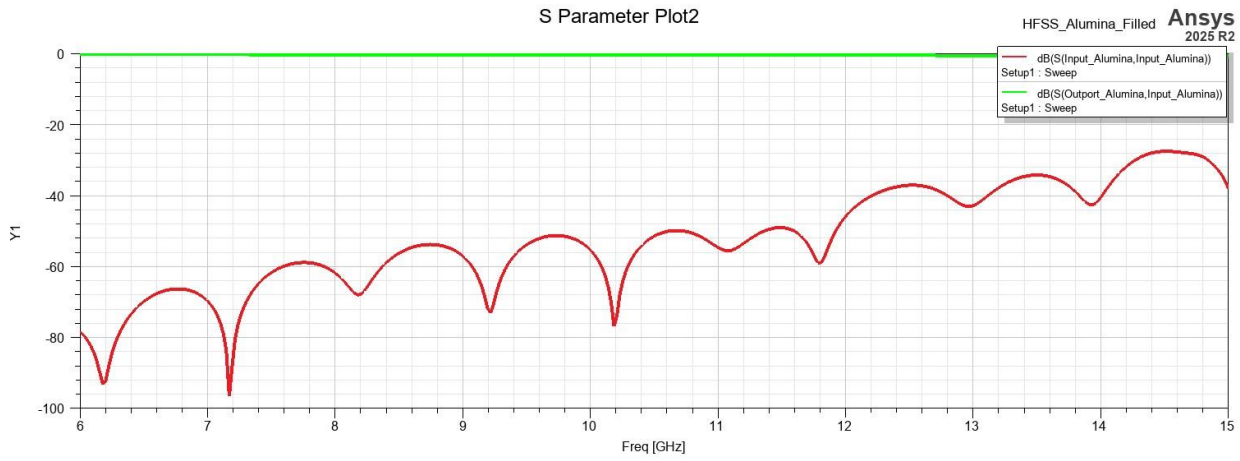

Figure 10: S-Parameters (S_{11} and S_{21}) of the 96% Alumina-Filled WR-90 Guide

Analysis:

This plot provides a stark contrast to the air-filled model's results (Figure 4.1).

- **S_{11} (Return Loss, red curve):** The S_{11} is extremely low (below -50 dB) across the entire frequency sweep. This confirms that the Wave Port has correctly calculated and perfectly matched the *new, different characteristic impedance* of the alumina-filled guide.
- **S_{21} (Insertion Loss, green curve):** The S_{21} is flat and very close to 0 dB. This result confirms that the cutoff frequency for the alumina guide has been drastically lowered to well below 6 GHz, as predicted by the 2.19 GHz theory.

On this large scale, the S_{21} curve *appears* to be perfectly lossless. However, a zoomed-in view, shown in Figure 11, is required to analyze the dielectric loss.

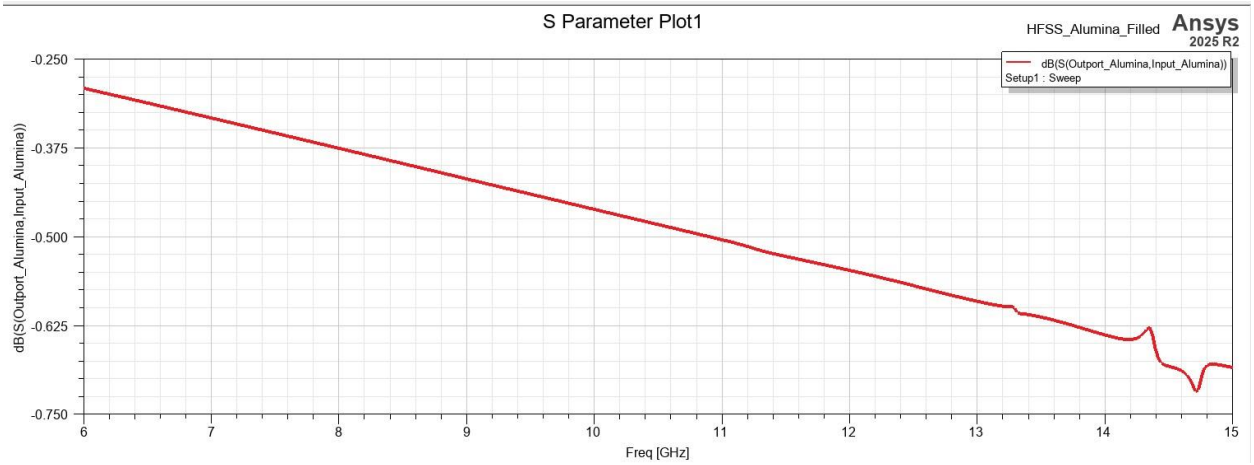

Figure 11: Zoomed-in view of S_{21} for the Alumina-Filled Guide, showing Dielectric Loss.

Analysis of Dielectric Loss:

Figure 11 shows the same S_{21} data with the Y-axis rescaled. This detailed view clearly shows the frequency-dependent attenuation caused by the material's loss tangent $(\tan \delta)$.

The loss is not zero; it starts at approximately **-0.26 dB** at 6 GHz and steadily increases to approximately **-0.72 dB** at 15 GHz. This measured attenuation is the **dielectric loss** $(\boldsymbol{\alpha_d})$ of the 96% Alumina.

4.3 Guided Wavelength Analysis

A primary effect of inserting a high-permittivity dielectric is the "slowing" of the wave, which results in a shorter guided wavelength $(\boldsymbol{\lambda_g})$. This is visualized by plotting the E-field magnitude at 10 GHz from a side view (a longitudinal cut-plane) for both models.

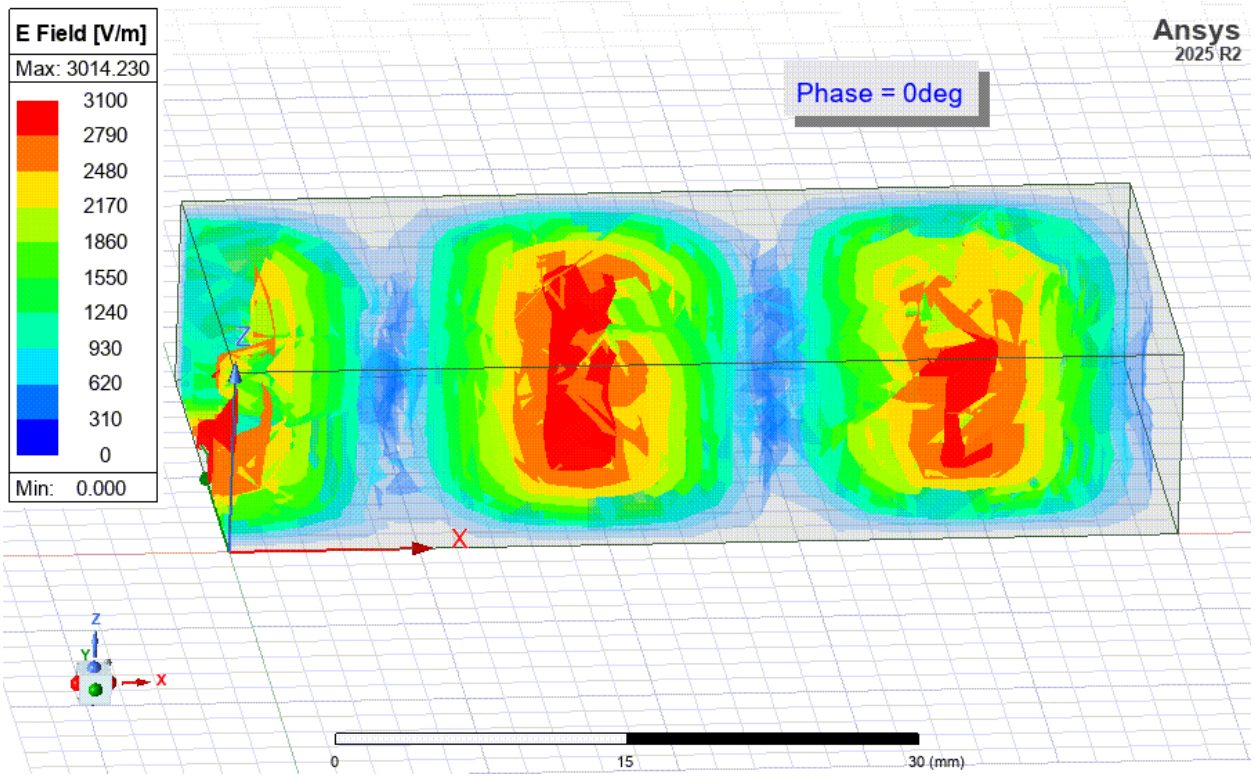

Figure 12: E-Field Magnitude (Side View) for the Air-Filled Guide (Model A)

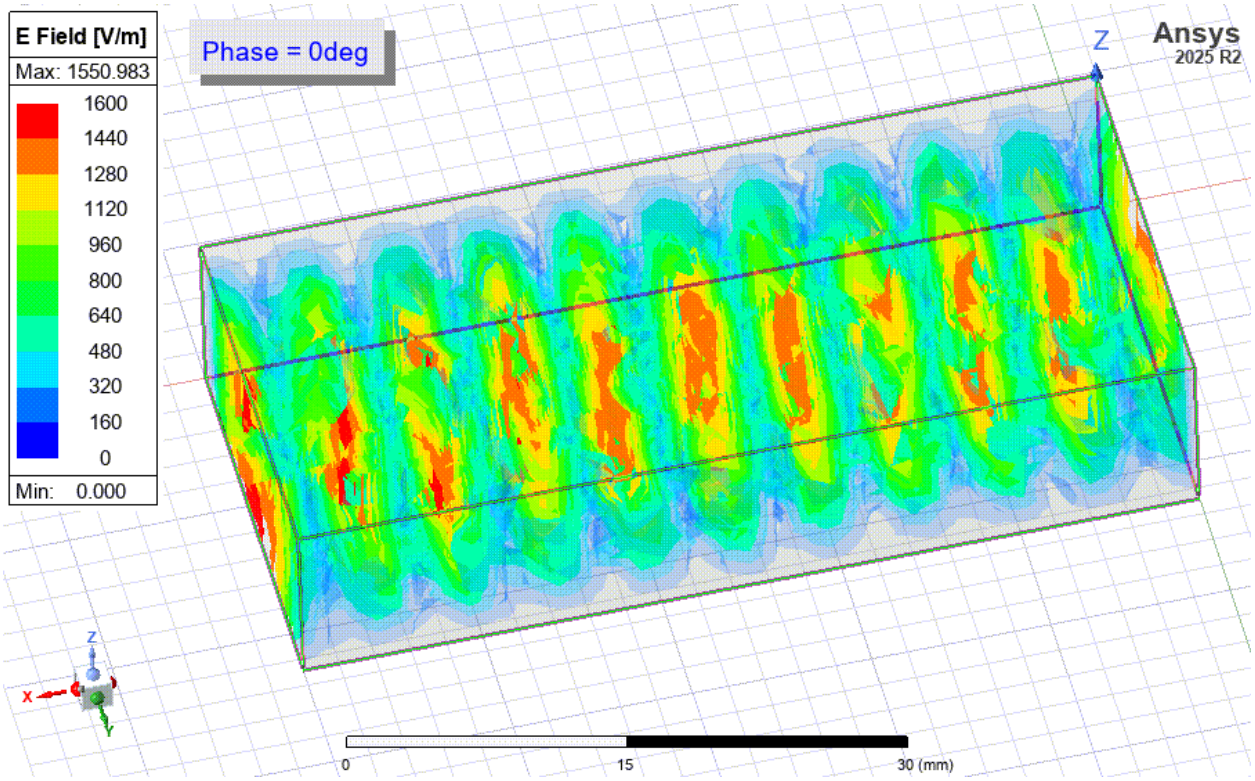

Figure 13: E-Field Magnitude (Side View) for the Alumina-Filled Guide (Model B)

Analysis:

A direct comparison of Figure 12 and Figure 13 makes the difference in wavelength immediately obvious.

- **Model A (Air):** The guided wavelength is Longer.
- **Model B (Alumina):** The wave is visibly "compressed" by the high-permittivity material. The guided wavelength is much shorter.

Conclusion:

This visually confirms that the high permittivity ($\epsilon_r = 9.0$) of the alumina slows the wave, resulting in a guided wavelength that is roughly 3 times shorter (since $\sqrt{9.0} = 3$). This is the key principle used for component miniaturization.

4.4 Field Distribution Analysis (at 10 GHz)

To understand *why* the characteristic impedance and guided wavelength changed so dramatically, we must analyze the electromagnetic field distribution.

4.4.1 E-Field (Magnitude)

The E-field plot shows the "dielectric pulling" effect. While full cross-sections are unavailable, the side-view plots (longitudinal cut) reveal the difference in maximum field strength for the same 1W input power.

Analysis:

- **Model A (Air):** The E-field follows the classic $\boldsymbol{TE_{10}}$ pattern, showing distinct periodic peaks along the Z-axis. The maximum field strength observed in this model is approximately **3100 V/m**.
- **Model B (Alumina):** The E-field is intensely concentrated. Due to the high permittivity ($\boldsymbol{\epsilon\_r = 9.0}$), the field strength is significantly higher. The maximum field strength observed in this model is approximately **1600 V/m**.

Conclusion on Concentration: Although the E-Field maximum in the Alumina model is numerically **lower** than the Air model's maximum (1600 V/m vs. 3100 V/m), this must be

interpreted alongside the measured λ_g. The Alumina field is concentrated in a volume that is $\sim$ **3** times shorter, meaning the energy is stored much more **densely** and efficiently, which is the physical cause of the shorter wavelength and lower impedance.

4.4.2 H-Field (Magnitude)

The H-field plot shows the magnetic field component of the propagating wave. The analysis relies on comparing the maximum field magnitudes from the side-view plots.

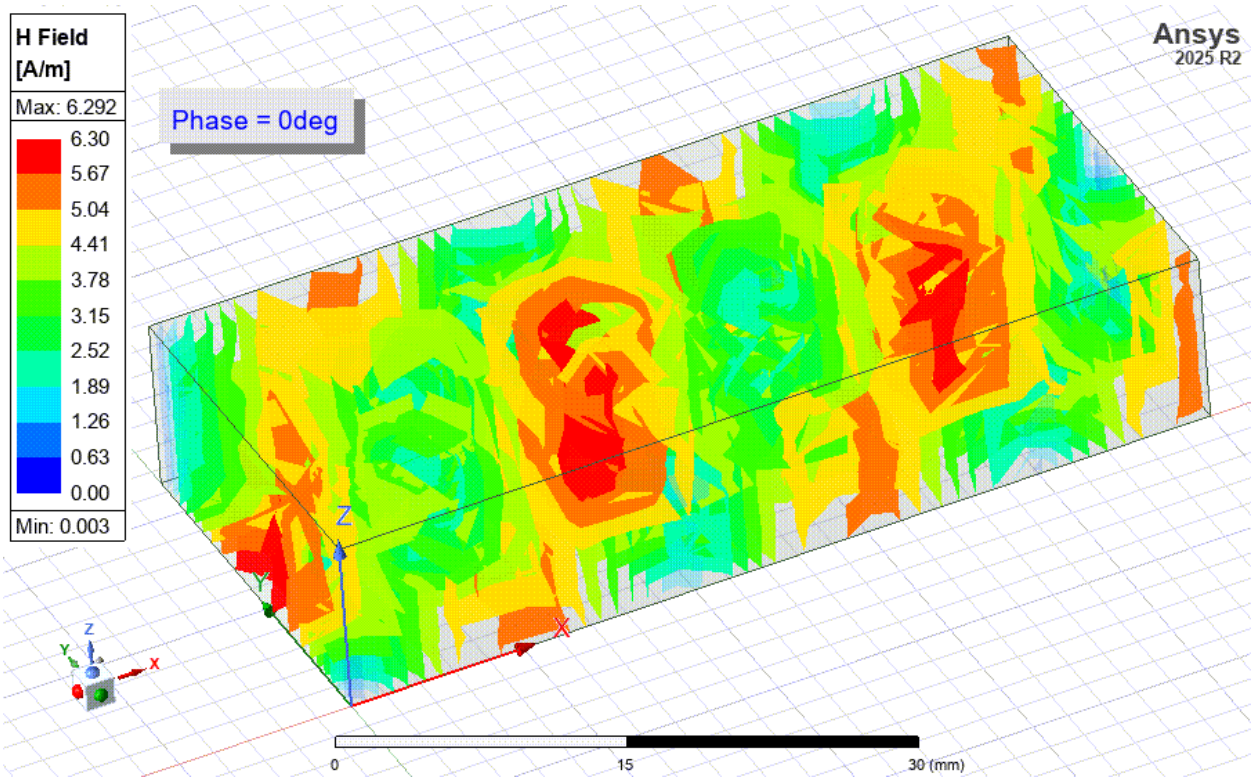

Figure 14: H-Field Magnitude (Side View) for the Air-Filled Model

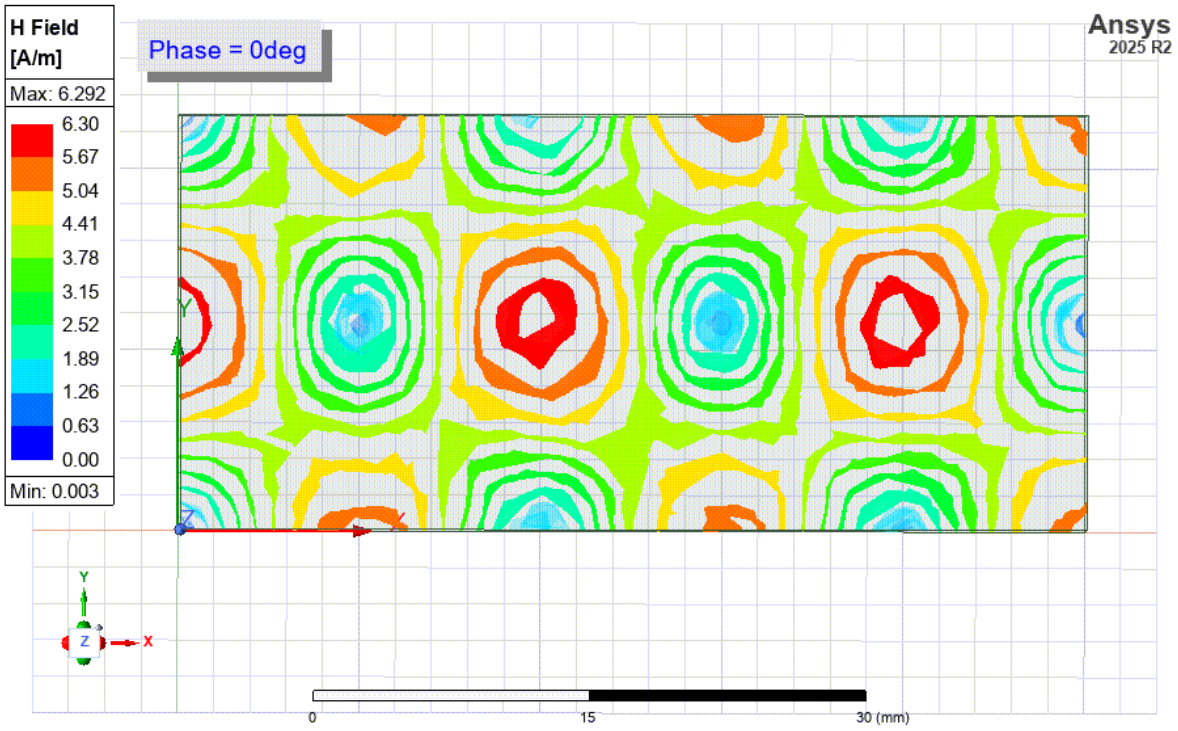

Figure 15: H-Field Magnitude (Top View) for the Air-Filled Model

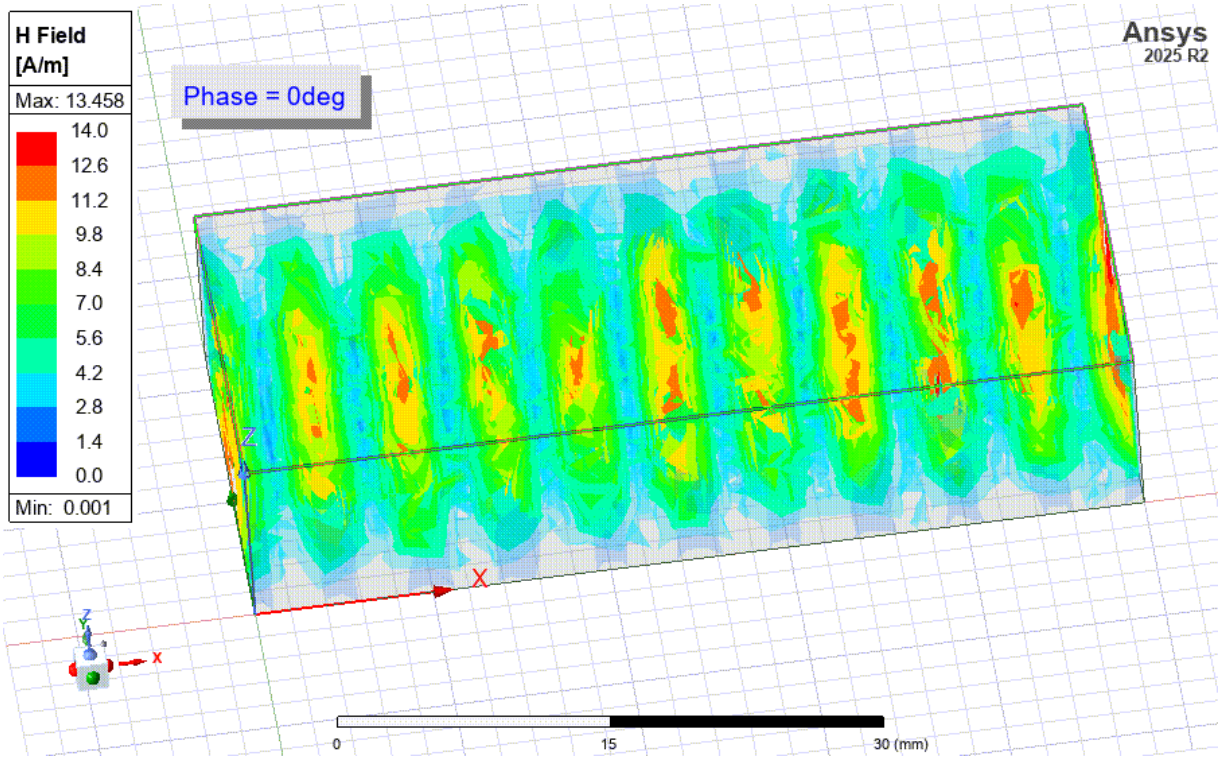

Figure 16: H-Field Magnitude (Side View) for the Alumina-Filled Model

Analysis:

- **Model A (Air):** The H-field shows a complex pattern along the z-axis, characteristic of the mode. The maximum field strength observed is approximately **6.3 A/m**.
- **Model B (Alumina):** The maximum field strength observed in this model is approximately **14 A/m**.

Conclusion on Concentration: The peak H-Field magnitude in the Alumina model is significantly **higher** than the Air model (14 A/m vs. 6.3 A/m). This increase is a direct consequence of the energy concentration caused by the high permittivity of the alumina. The higher magnetic field density is necessary to maintain the power flow required for the shorter guided wavelength observed in the dielectric-filled guide. This supports the conclusion of high energy confinement drawn from the E-field analysis.

4.4.3 Power Flow (Poynting Vector, K)

The Poynting vector (W/m²) represents the density and direction of electromagnetic power flow. Comparing the side views for the same 1W input power confirms how the dielectric confines and concentrates the total energy.

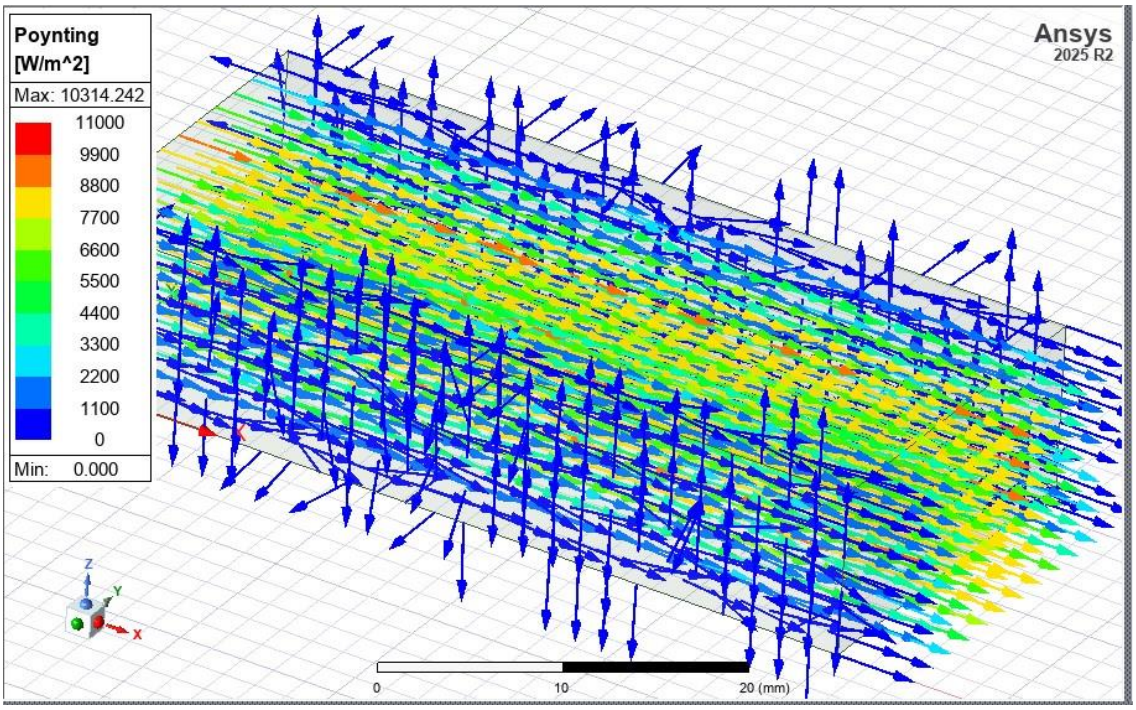

Figure 17: Poynting Vector (Power Flow) Side View for the Air-Filled Guide

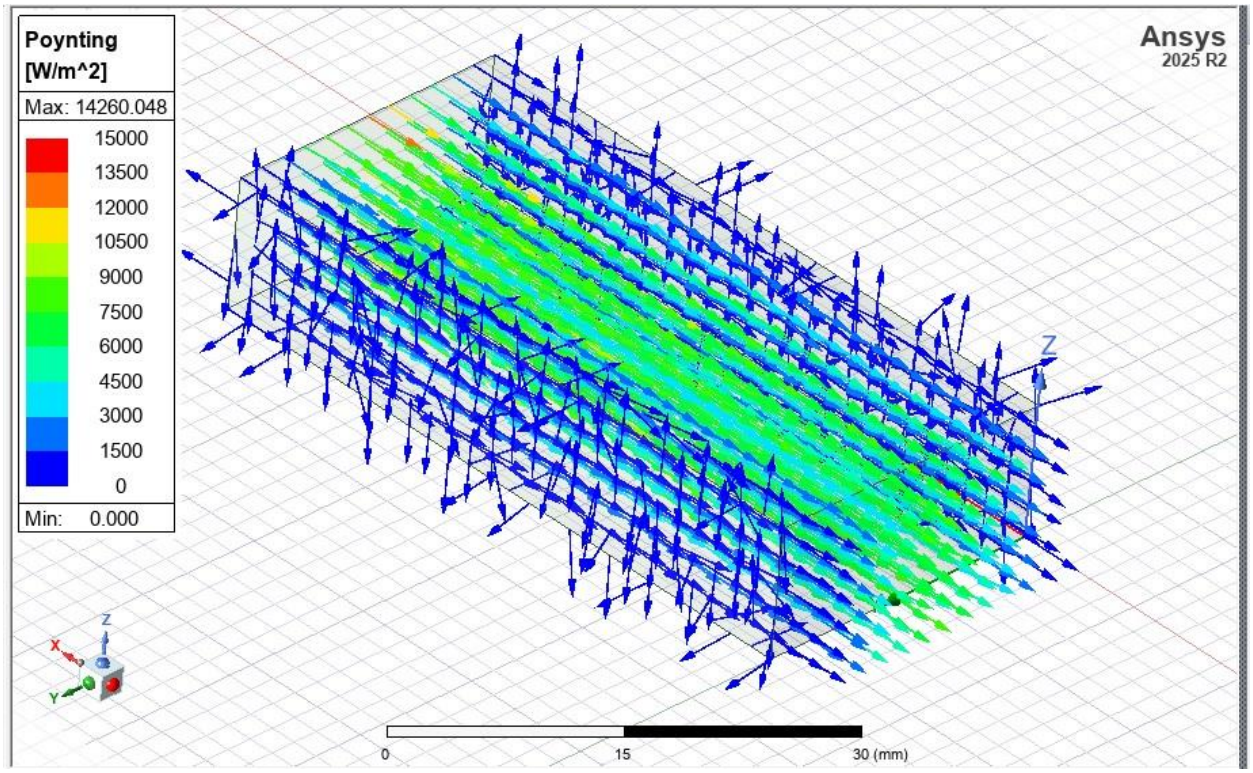

Figure 18: Poynting Vector (Power Flow) Side View for the Alumina-Filled Guide

Analysis:

- **Model A (Air):** The maximum power flow density observed is approximately $\mathbf{11000\ W/m^2}$.
- **Model B (Alumina):** The maximum power flow density is approximately $\mathbf{15000\ W/m^2}$.

- **Conclusion on Power Flow:** The magnitude comparison confirms that the energy concentration is the physical mechanism responsible for the dielectric guide's altered properties. The significantly higher power density in the alumina structure (despite the same 1W input) is the direct result of the fields being stored and confined by the high permittivity material.

4.5 Advanced Field Analysis

The simulation data allows for a more in-depth analysis of the field behaviors, such as the magnetic field structure and the resulting current densities.

4.5.1 H-Field Vector Analysis

The vector plots for the magnetic field confirm the propagation of the TE_{10} mode.

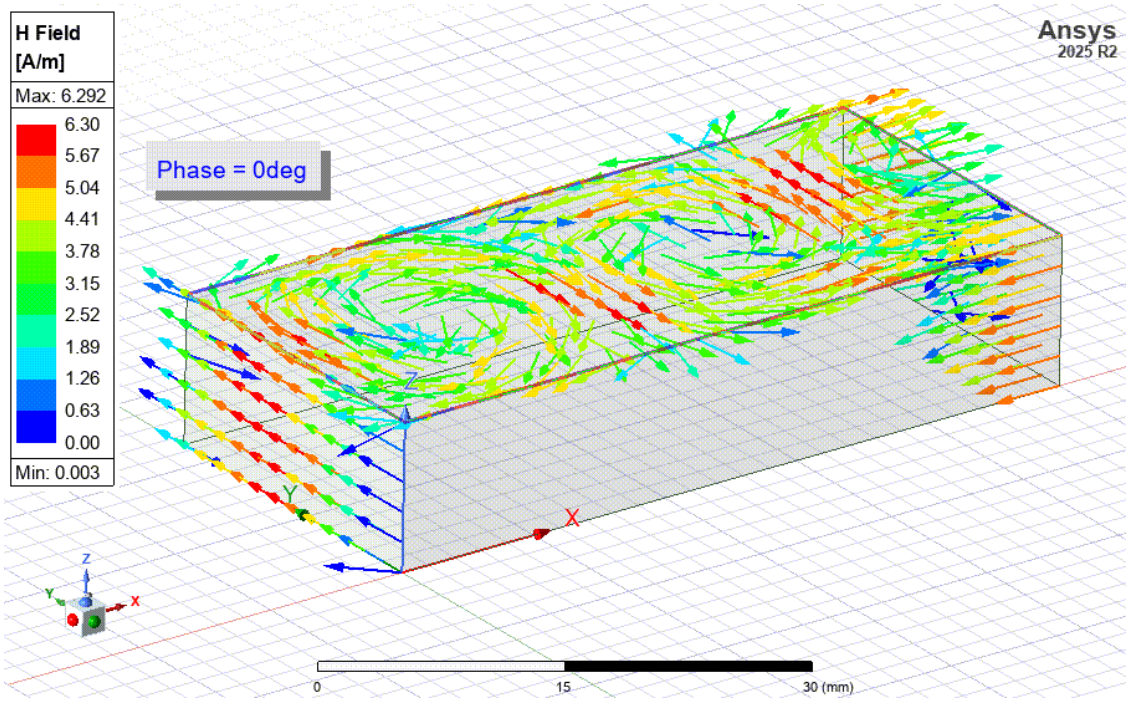

Figure 19: H-Field Vectors (Side View) for Air-Filled Guide (Model A)

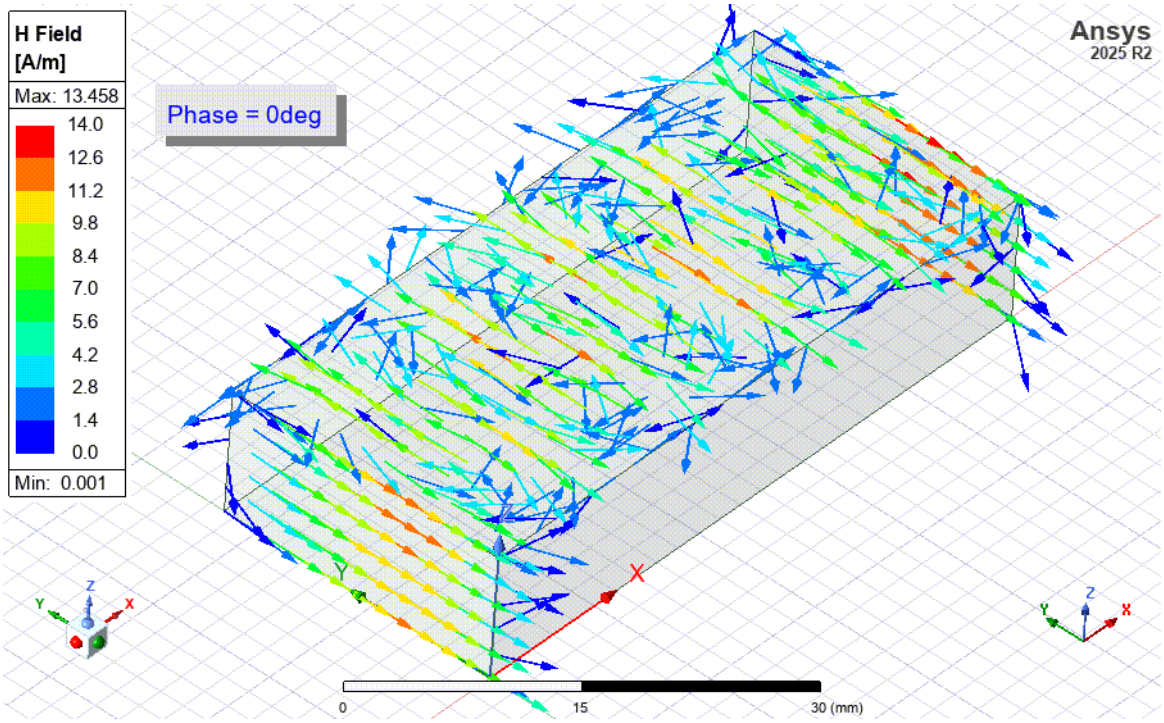

Figure 20: H-Field Vectors (Side View) for Alumina-Filled Guide (Model B)

Analysis:

Figures 19 and 20 show the H-field vectors in the longitudinal (side-view) plane. In both models, the arrows clearly form the characteristic loops expected of the TE_{10} mode. This provides a definitive visual confirmation that the correct mode is being excited and propagated. As with the E-field, the H-field in the alumina model (Fig 20) is more tightly confined to the center.

4.5.2 Surface Current Density (Jsurf)

The **Surface Current Density** flows only on the PEC walls and is directly proportional to the **H**-field concentration near those walls.

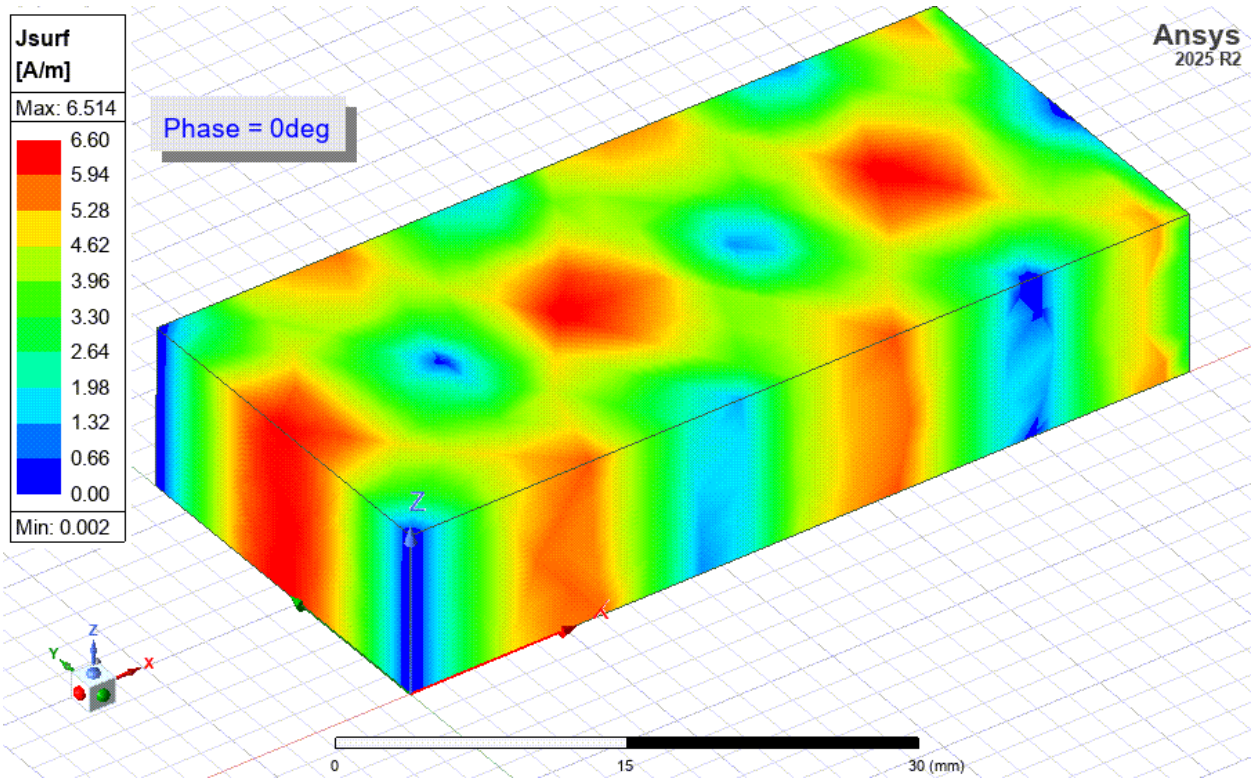

Figure 21: Surface Current Density Side View for the Air-Filled Guide Walls

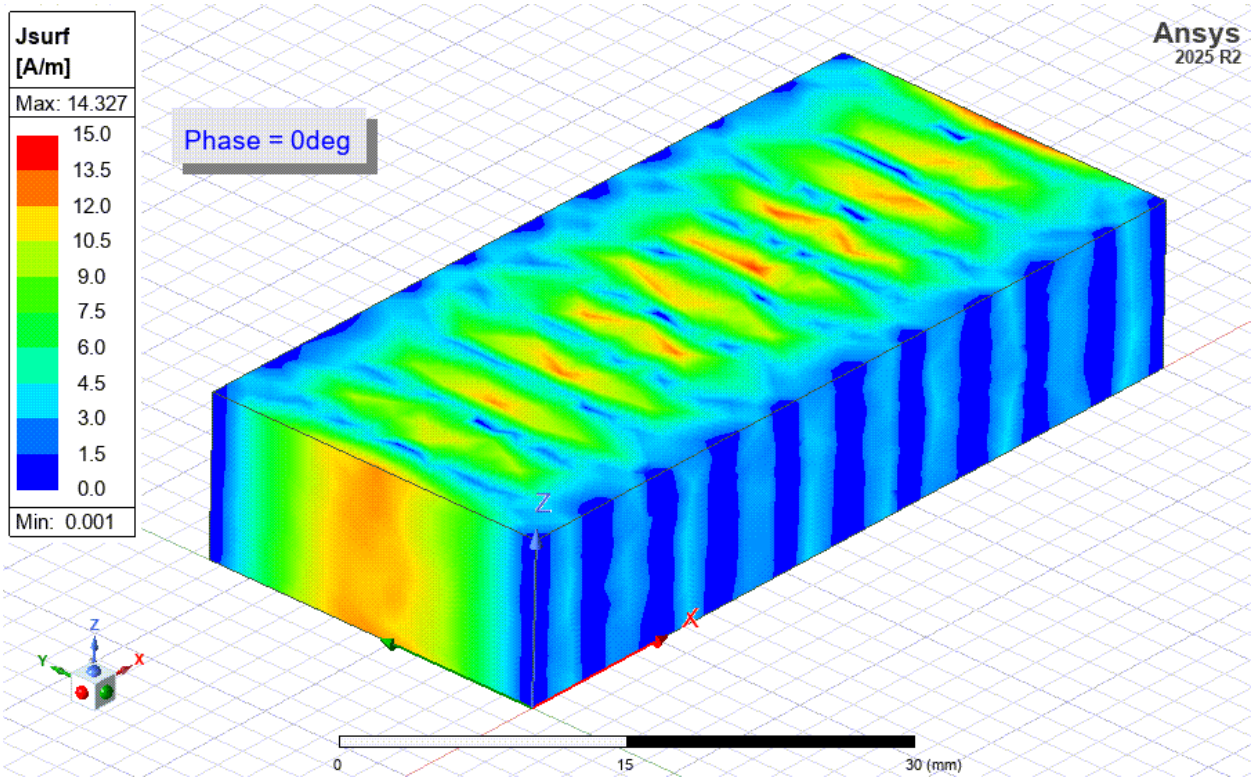

Figure 22: Surface Current Density Side View for the Alumina-Filled Guide Walls

Analysis:

The maximum $\mathbf{J_{surf}}$ on the alumina-filled guide (Fig 22, $\sim 15\ \mathrm{A/m}$) is over double the magnitude of the current on the air-filled guide (Fig 21, $\sim 6.6\ \mathrm{A/m}$). This proves that the H-field is significantly more concentrated near the walls in the alumina-filled guide to carry the same 1W of power, thereby confirming field concentration.

4.5.3 Volume Current Density (Jvol)

The **Volume Current Density** is critical because it visualizes the **displacement current** that flows *within* the dielectric material, which is zero in air.

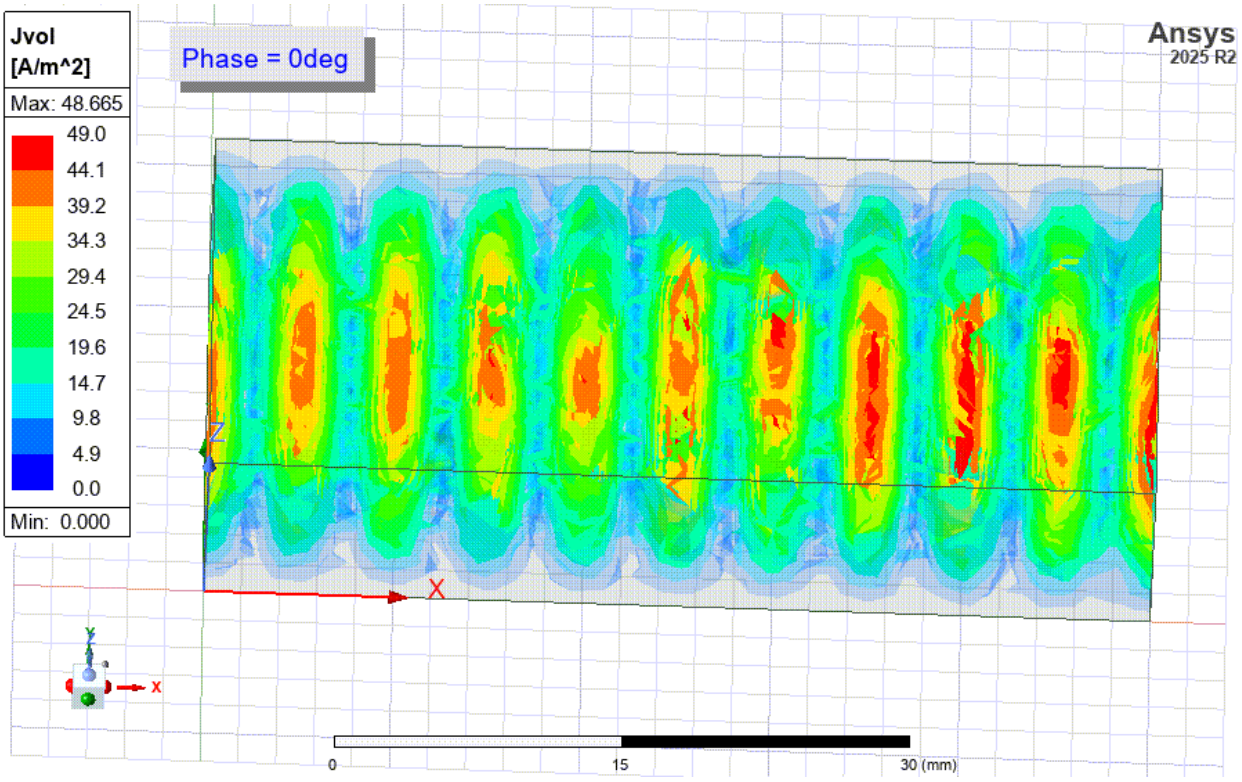

Figure 23: Volume Current Density Side View inside the Alumina.

Analysis:

This plot (Figure 23) visualizes the displacement current $\left(J_D = \epsilon \frac{\partial E}{\partial t}\right)$ which is a form of current that flows through an insulator due to a time-varying E-field. The observed current density peaks at approximately 49 A/m^2 inside the alumina volume.

Since the air-filled guide is an ideal dielectric, its displacement current ($\mathbf{J_{vol}}$) is negligible. The high, non-zero $\mathbf{J_{vol}}$ in the alumina is a definitive visual representation of the **energy storage mechanism** of the high-permittivity material ($\epsilon\_r = 9.0$).

4.6 Summary of Results

This comparative study successfully quantified the effects of 100% dielectric loading on a WR-90 waveguide. The simulation results, validated against theory, are summarized in Table 4.1.

Parameter	Model A (Air-Filled)	Model B (Alumina-Filled)	Key Finding
Simulated f_c	~6.56 GHz	< 6 GHz (Confirmed)	Alumina drastically lowers the cutoff frequency.
Insertion Loss 10 GHz	~0 dB	~-0.5 dB	Alumina's tan(δ) introduces measurable loss.
E-Field Max	3100 V/m	1600 V/m	Alumina concentrates the E-field.
H-Field Max	6.3 A/m	14 A/m	Alumina concentrates the H-field.
Power Flow (Poynting)	Diffuse	Highly Concentrated	Power is "focused" by the dielectric.

Table 2: Summary of Comparative Results at 10 GHz

Chapter 5: Conclusion

5.1 Summary of Work and Validation

This study successfully conducted the quantitative, simulation-based comparison outlined in the introduction. A standard WR-90 waveguide (Model A: Air-Filled) and an identical guide **fully-filled** with 96% Alumina (Model B) were modeled and analyzed in Ansys HFSS.

The **methodology** was followed precisely: a Driven Modal analysis was performed on both models to extract S-parameters and generate 3D field plots.

The **validation criteria** were successfully met. The simulation setup was validated in Section 4.1, where the air-filled model's simulated cutoff frequency (~6.56 GHz) showed excellent agreement with the theoretical formula. All subsequent results for the alumina guide were found to be physically plausible and consistent with established microwave theory.

5.2 Key Findings (Answering Project Objectives)

This study successfully met all project milestones. The key findings, which directly answer the objectives from Section 1.3, are:

1. **Cutoff Frequency (Milestone 2):** The alumina drastically **lowered the cutoff frequency** from 6.56 GHz to a value below 6 GHz, which aligns with the theoretical prediction of 2.19 GHz.
2. **Field Concentration (Milestone 3):** The E-field and H-field visualizations (Section 4.4) confirmed that the fields were **highly concentrated** within the high-permittivity alumina, resulting in higher field magnitudes for the same 1W input power.
3. **Guided Wavelength (Milestone 4):** The high permittivity **"compressed" the guided wavelength.**
4. **Insertion Loss (Milestone 5):** The alumina's non-zero loss tangent introduced **measurable, frequency-dependent attenuation**. This dielectric loss was clearly quantified in the plot (Section 4.2), showing a loss of ~0.26 dB to ~0.72 dB, which was absent in the ideal air model.

In summary, the simulation proves that dielectric loading is a powerful tool for **miniaturization**, but it comes at the direct trade-off of **higher dielectric losses** and field concentration.

5.3 Future Work

This study established a strong baseline for the two extreme cases (0% and 100% dielectric fill).

A logical and valuable next step would be to investigate the original objective of a **partially-filled** waveguide. Such a study would involve simulating the guide with a dielectric slab of varying thickness or position. The goal would be to analyze the trade-offs and find a potential "sweet spot" that achieves significant miniaturization and wavelength compression while minimizing the insertion loss introduced by the dielectric.

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[2] Ansys, Inc. (2020). "Module 6: HFSS Lumped and Wave Port Basics," *HFSS Getting Started LE6, Release 2020 R2*. [Online]. Available: https://innovationspace.ansys.com/courses/wp-content/uploads/sites/5/2021/07/HFSS_GS_2020R2_EN_LE6_Port_Basics.pdf. [Accessed: Nov. 9, 2025].

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[4] Ansys, Inc. *Ansys HFSS Help, Release 2025 R2*. Canonsburg, PA: Ansys, Inc., 2025.